Testing the TMD framework at hadron colliders

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- Azimuthal asymmetries in support of TMD framework
- BM effect ($h_1^\perp$) $+$ its asymmetries
- Sivers effect ($f_{1T}^\perp$) $+$ its asymmetries
- Support from lattice QCD
- Linearly polarized gluons
- Spin effects in fragmentation
- Energy scale dependence

Motivation

Azimuthal asymmetries have been measured in various processes, which are most naturally described by including partonic transverse momentum distributions (TMDs)

Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98

Such transverse momentum dependence can be correlated with the spin dependence

spin-orbit correlations

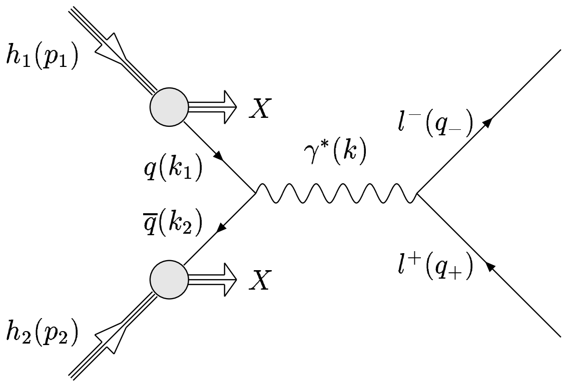
One can experimentally test the TMD framework, which is based on TMD factorization

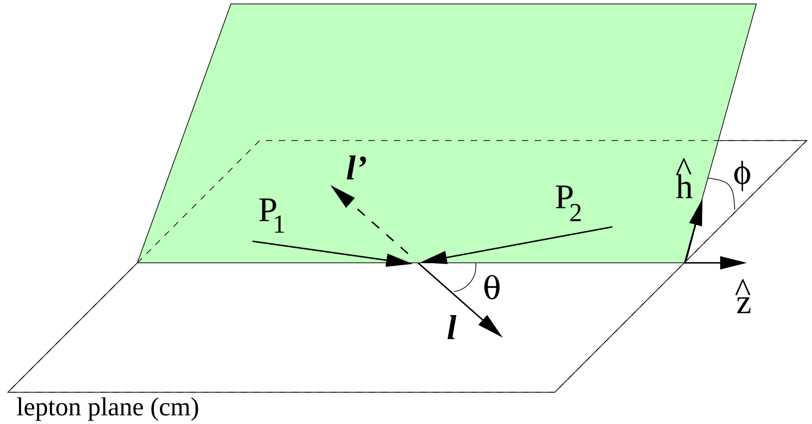
Collins & Soper '81; Ji, Ma & Yuan '04 & '05

First let's have a look at the current experimental support for this description

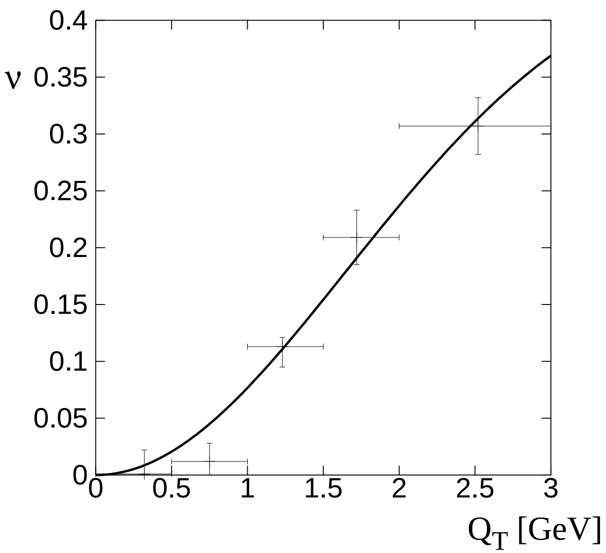
1) azimuthal asymmetries in unpolarized scattering
2) azimuthal asymmetries in polarized scattering

Azimuthal asymmetries





$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta\, \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$



Unpolarized $\pi^- D(W) \to \mu^+\mu^- X$ DY data

$\sqrt{s} \approx 20 \pm 3$ GeV

lepton pair invariant mass $Q \sim 4 - 12$ GeV

NA10 Collab. ('86/'88) & E615 Collab. ('89)

Quark polarization inside unpolarized hadrons

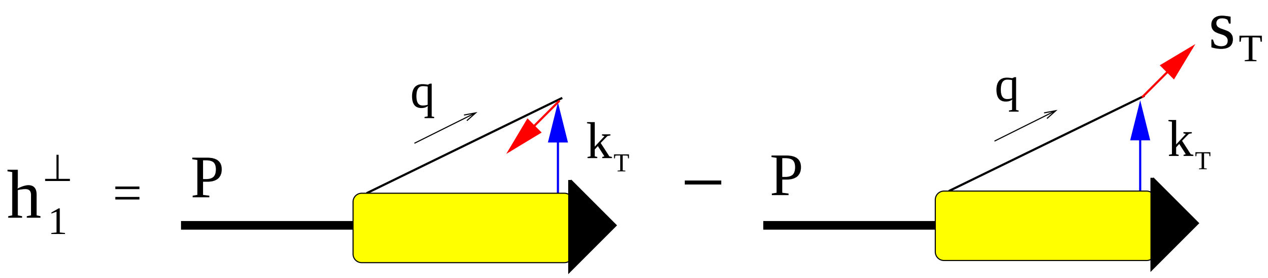
Unpolarized $\pi^- D(W) \to \mu^+\mu^- X$ DY data incompatible with *collinear* pQCD

Collins '79; Brandenburg, Nachtmann & Mirkes '93; Mirkes & Ohnemus '95

Naturally explained within the TMD framework in terms of $h_1^\perp$

D.B., '99

Transversely polarized quarks inside an *unpolarized* hadron

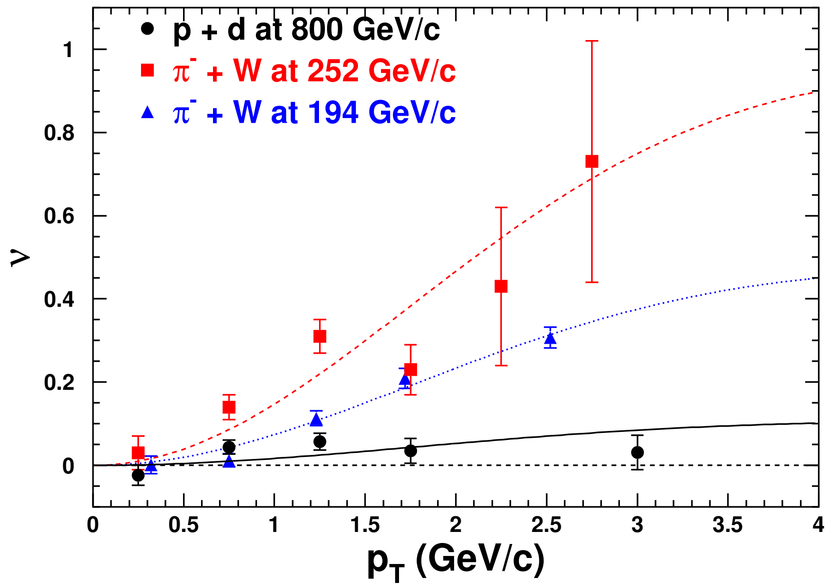


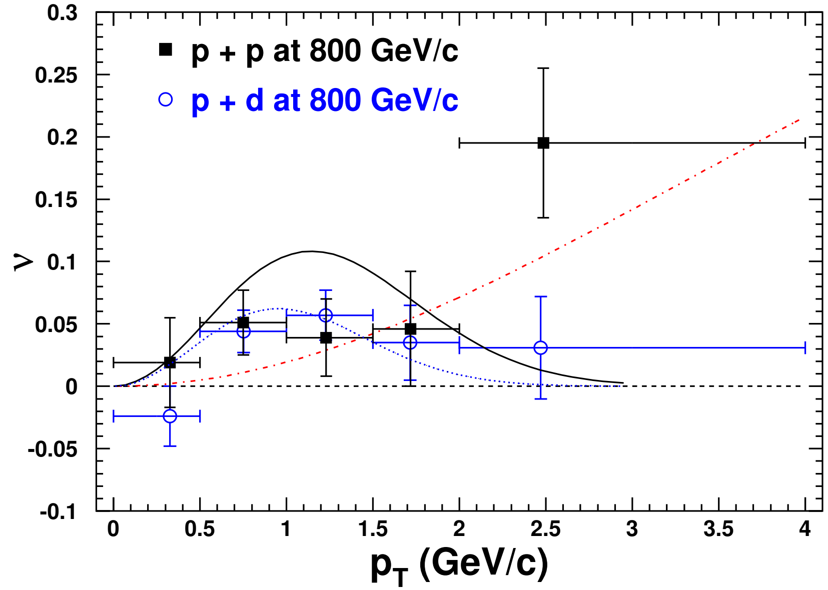
D.B. & Mulders '98

$h_1^\perp$ generates azimuthal asymmetries in unpolarized collisions, such as the $\cos 2\phi$ in DY

Recent unpolarized DY data

Asymmetry for $p\,p$ and $p\,d$ expected to be smaller, as confirmed by recent Fermilab data



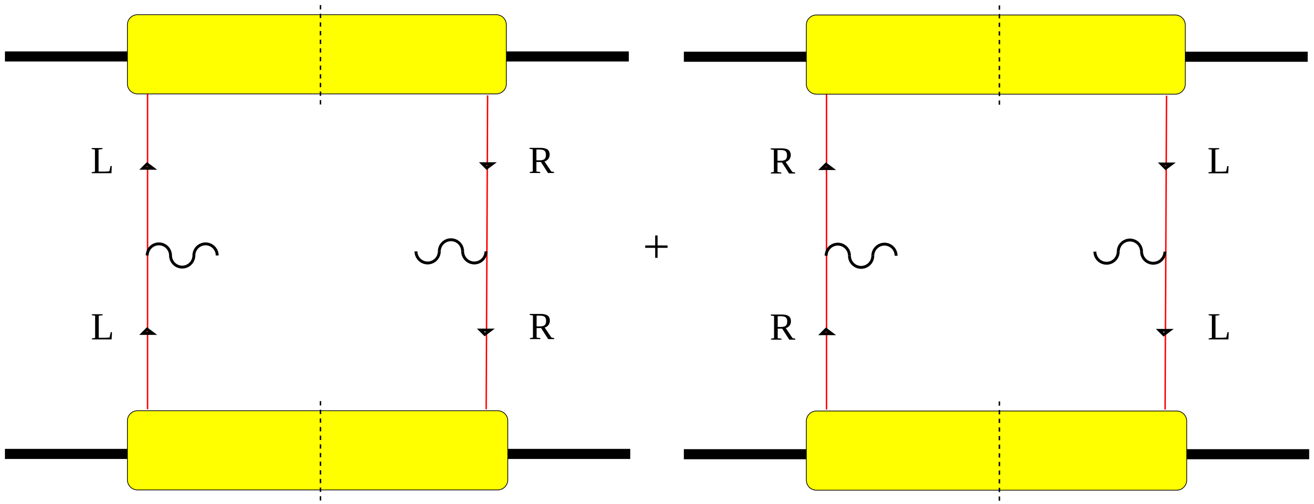


FNAL-E866/NuSea Collaboration, L.Y. Zhu *et al.* '07 & '09

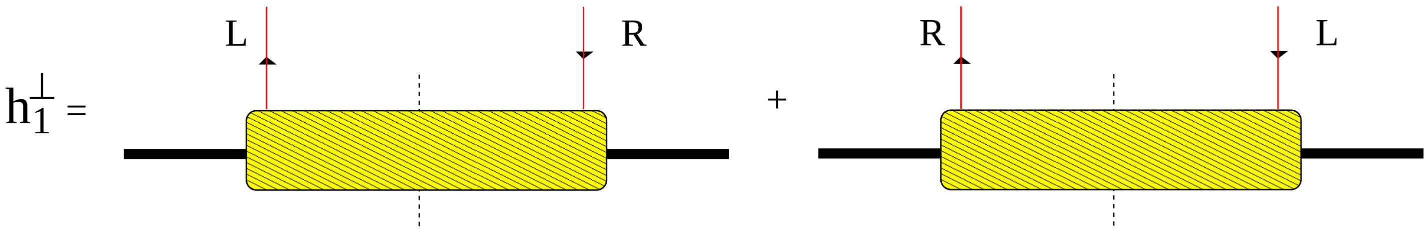
Natural to expect $\nu(p\,p) \ll \nu(\bar{p}\,p) \approx \nu(\pi\,p)$, due to valence antiquarks in $\bar{p}$ and π

Angular asymmetry requires helicity flip

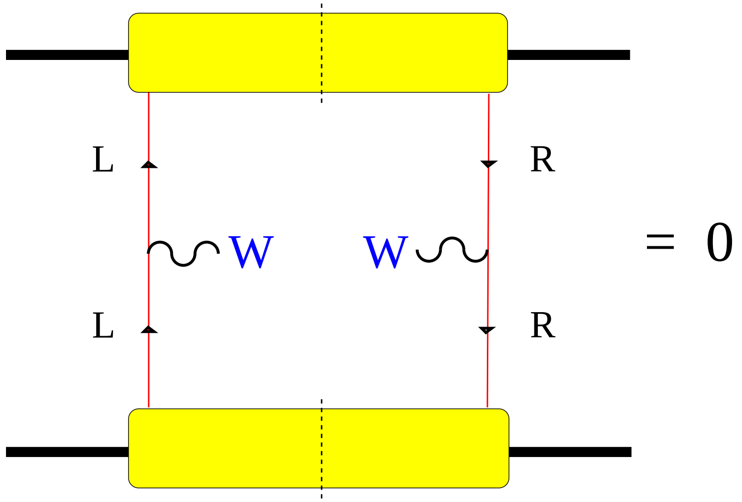
A $\cos 2\phi$ asymmetry arises from an interference between $+1$ and -1 photon helicities

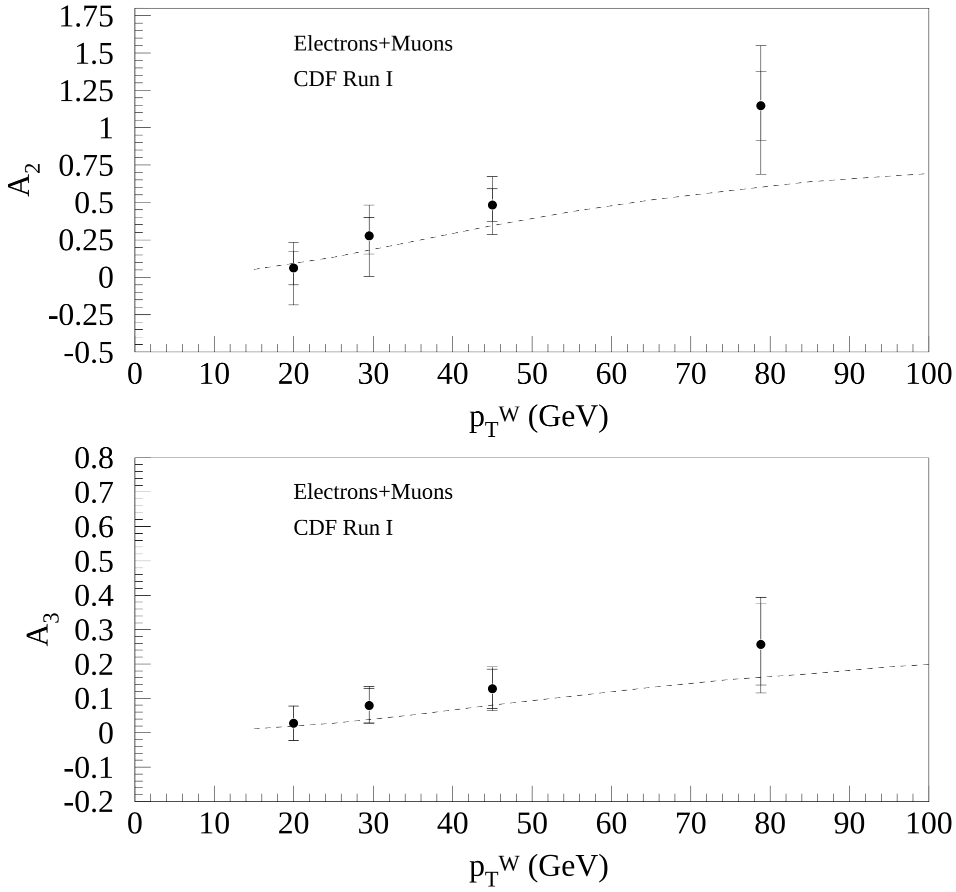


Perturbatively very small, but nonperturbatively it may be large (χSB)



No deviation from collinear pQCD in W production





W couples only to lefthanded quarks

$\bar{p}\,p \to W\,X \to \ell\,\nu_\ell\,X$ data consistent with NLO pQCD prediction

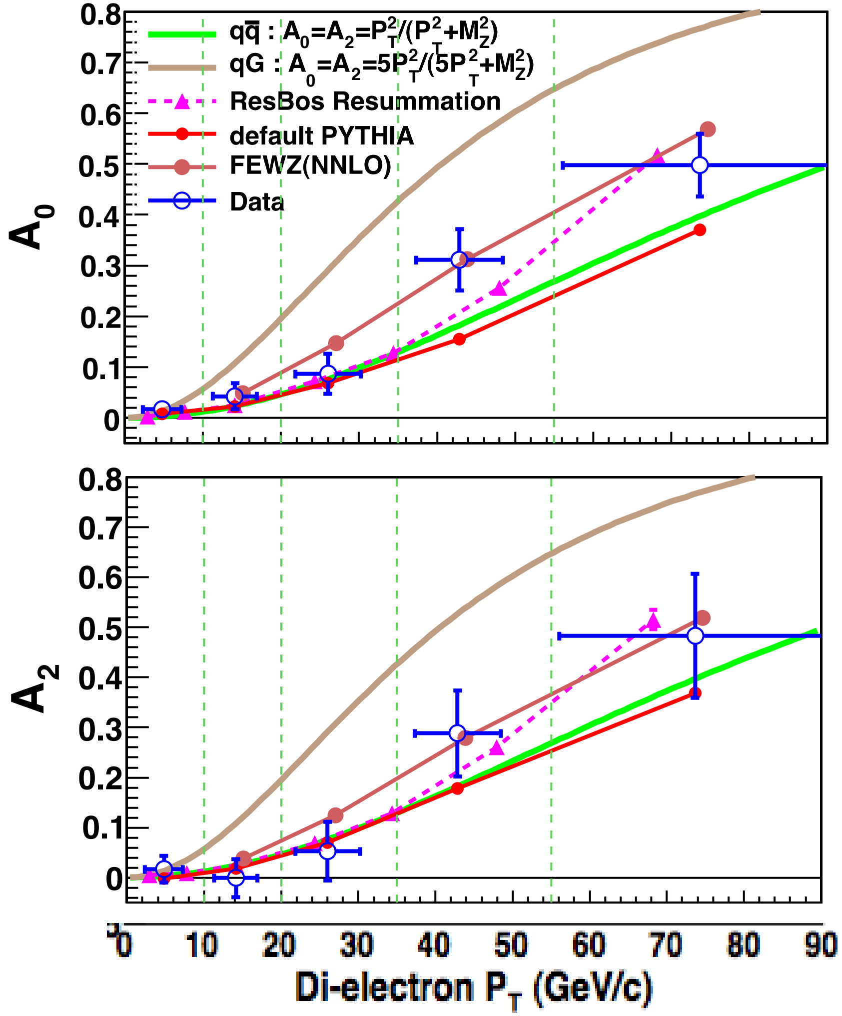
Mirkes, Ohnemus, PRD 50 (1994) 5692

No need/room for extra contributions

$A_2 \to \cos 2\phi$ and $A_3 \to \cos\phi$

CDF Collab., D. Acosta *et al.*, PRD 73 (2006) 052002

No deviation from collinear pQCD in Z production



$\cos 2\phi$ asymmetry (A_2) in

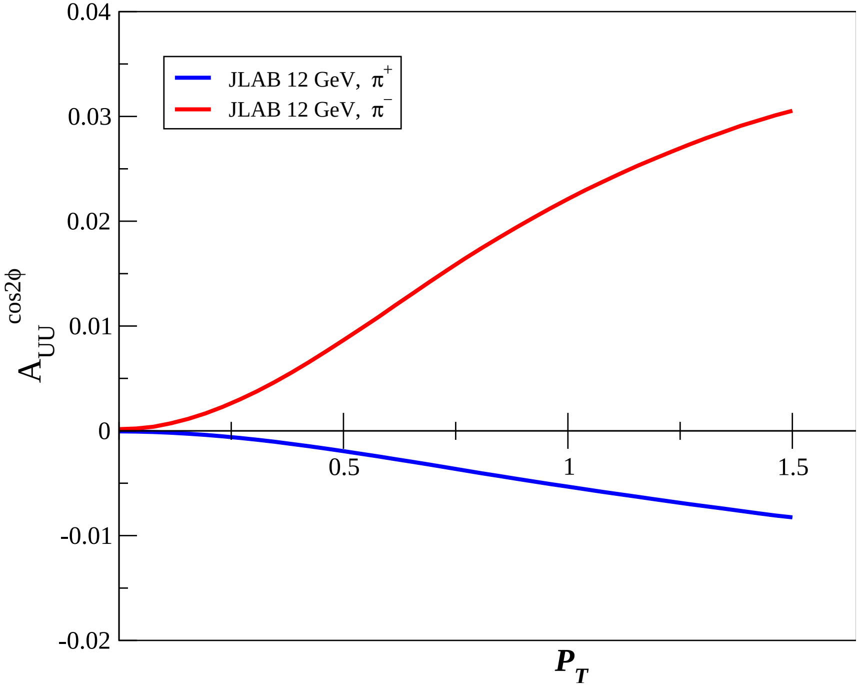
$p\,\bar{p} \to Z \to e^+e^-X$ at $\sqrt{s} = 1.96$ TeV

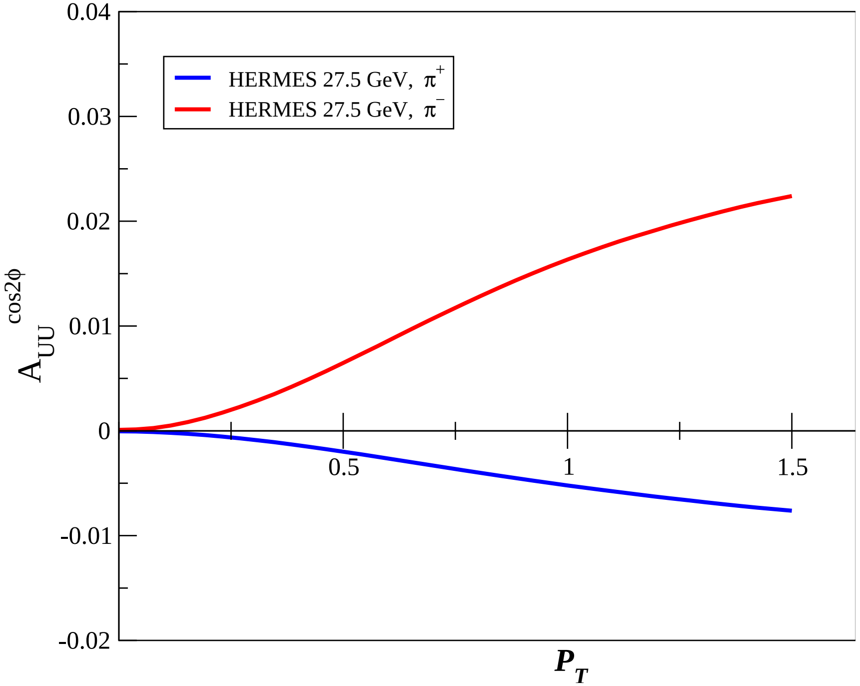
CDF Collaboration, arXiv:1103.5699

Too high energy and too high p_T to expect major contribution from $h_1^\perp$

$h_1^\perp$ in unpolarized SIDIS

$\cos 2\phi$ asymmetry in $e\,p \to e'\,\pi\,X \propto h_1^\perp H_1^\perp$ ($H_1^\perp$ Collins fragmentation function)





Gamberg, Goldstein & Schlegel, PRD 77 (2008) 094016

See also: Barone, Lu & Ma, PLB 632 (2006) 277

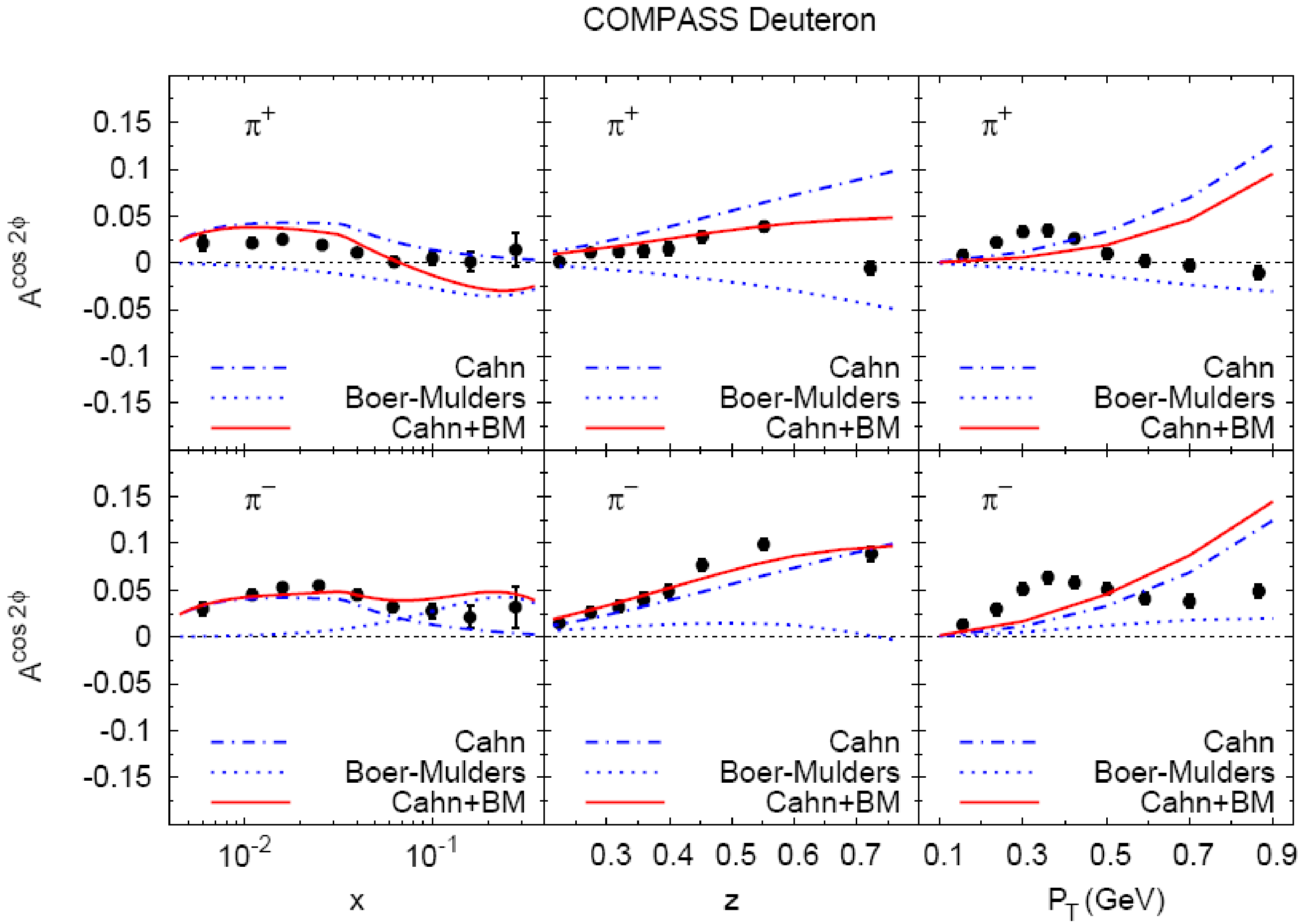
Recent measurements: CLAS Collaboration, PRD 80 (2009) 032004

COMPASS Collaboration, e.g. arXiv:0907.5511

HERMES Collaboration, e.g. arXiv:0901.2438

$h_1^\perp$ in unpolarized SIDIS at low Q^2

The problem is that at low Q^2 the twist-4 Cahn effect also enters, for example:

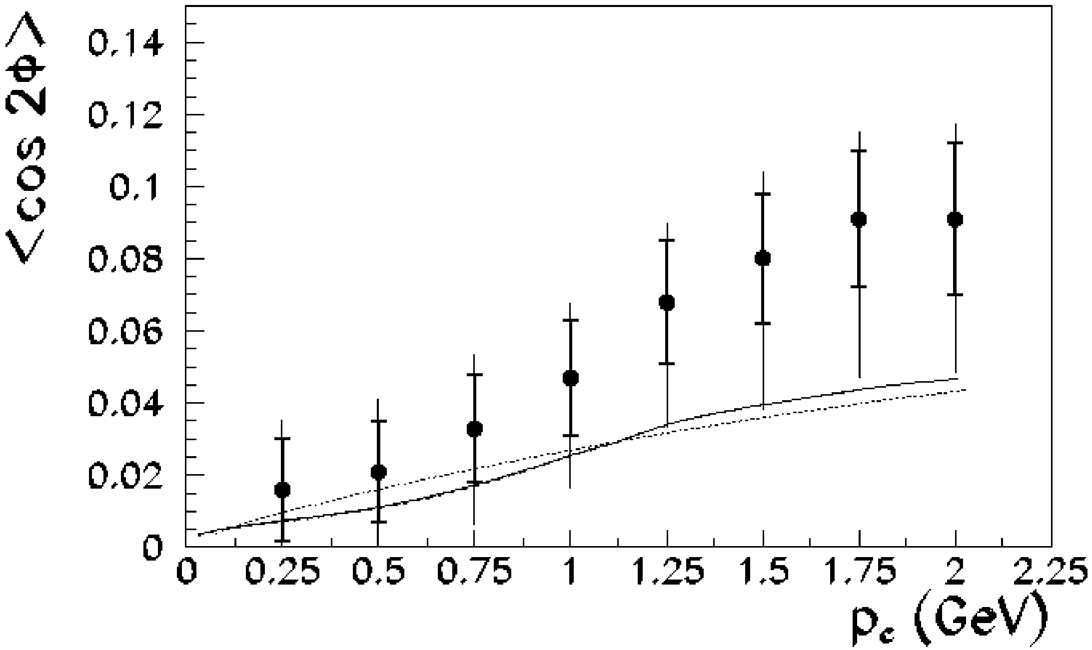


Barone, Melis, Prokudin, arXiv:0912.5194

$h_1^\perp$ in unpolarized SIDIS at high Q^2

ZEUS data for charged hadrons at $\langle Q^2 \rangle = 750$ GeV^2 is consistent with pQCD

ZEUS Collaboration, PLB 481 (2000) 199 & EPJC 51 (2007) 289



Note: p_c is a lower cut on observed p_T

Cahn effect is negligible at high Q^2 ($\sim M^2/Q^2$), but other contributions also fall off

The perturbative high-Q_T contribution falls off as Q_T^2/Q^2 ($Q_T^2 \gg M^2$)

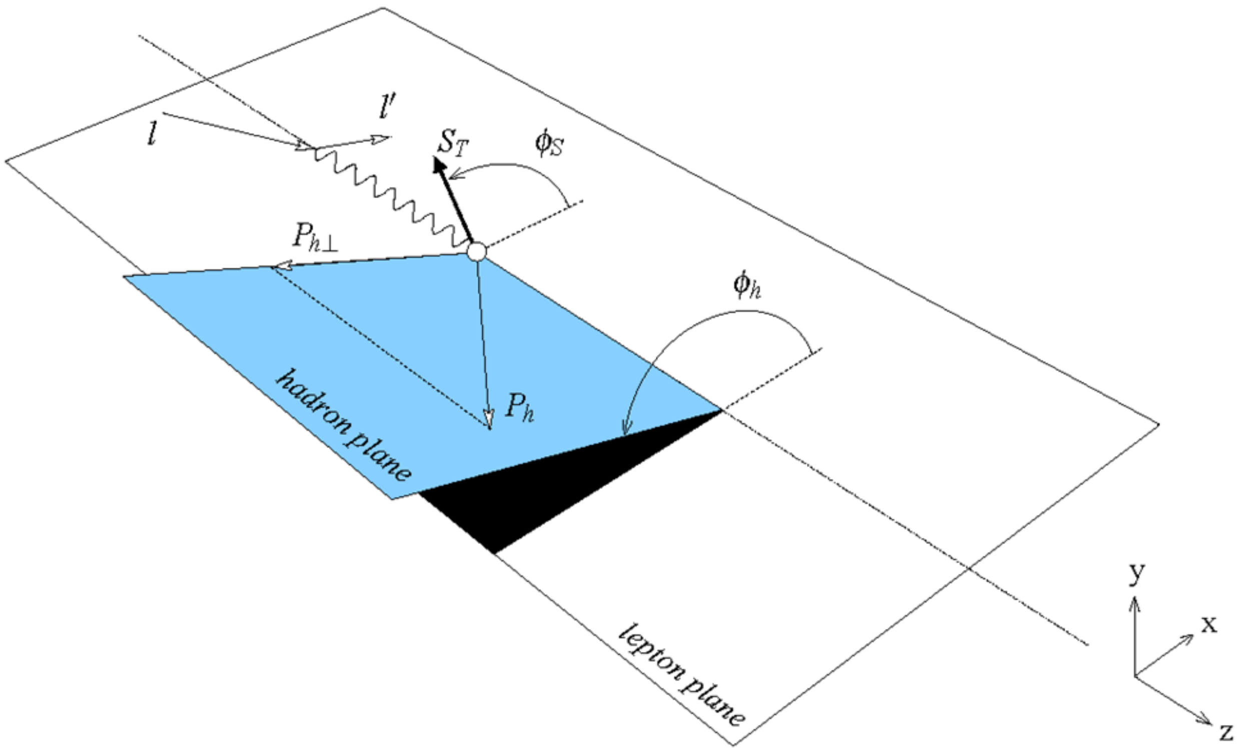
Azimuthal *spin* asymmetries

Spin dependent azimuthal asymmetries, such as $\sin(\phi_h \pm \phi_S)$ asymmetries, have been clearly observed in semi-inclusive DIS (SIDIS): $e\,p^\uparrow \to e'\,h\,X$

HERMES Collab, PRL 2009; COMPASS, PLB 2010

SIDIS

$e\,p \to e'\,h\,X$

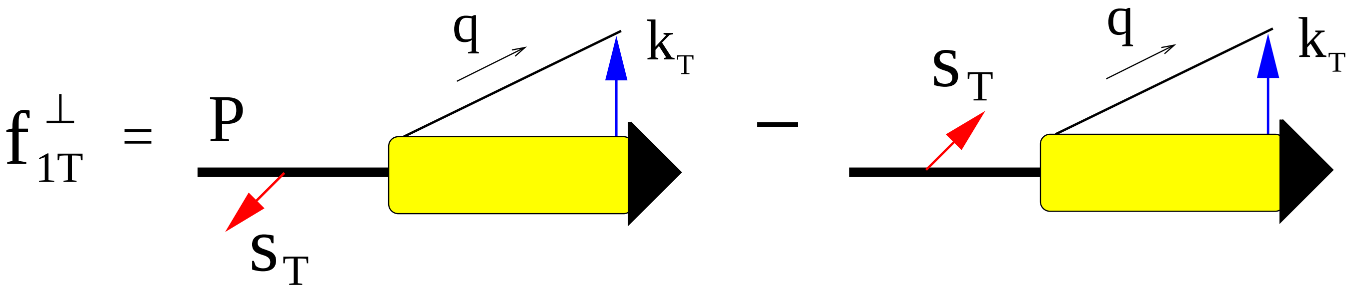


Can be explained by the Sivers ($f_{1T}^\perp$) and Collins ($H_1^\perp$) effects

Sivers '90; Collins '93

Sivers effect

The Sivers effect ('90) is described by a $\boldsymbol{k}_T$ *and* $\boldsymbol{S}_T$ dependent distribution function



The Sivers function gives the size of a $\boldsymbol{P}\cdot(\boldsymbol{k}_T \times \boldsymbol{S}_T)$ correlation, which is T-odd since under time reversal transformation: $\boldsymbol{P} \to -\boldsymbol{P}$ and $\boldsymbol{S} \to -\boldsymbol{S}$

Effect often referred to as "naive" T-odd, as time reversal also interchanges $i \leftrightarrow f$

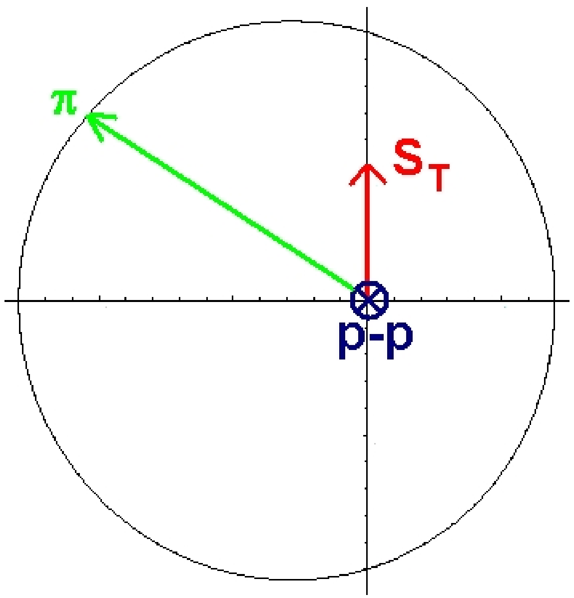
Proposed to explain data on $p^{\uparrow} + p \to \pi^0 + X$ at $\sqrt{s} \approx 7$ GeV (Antille *et al.* '80)

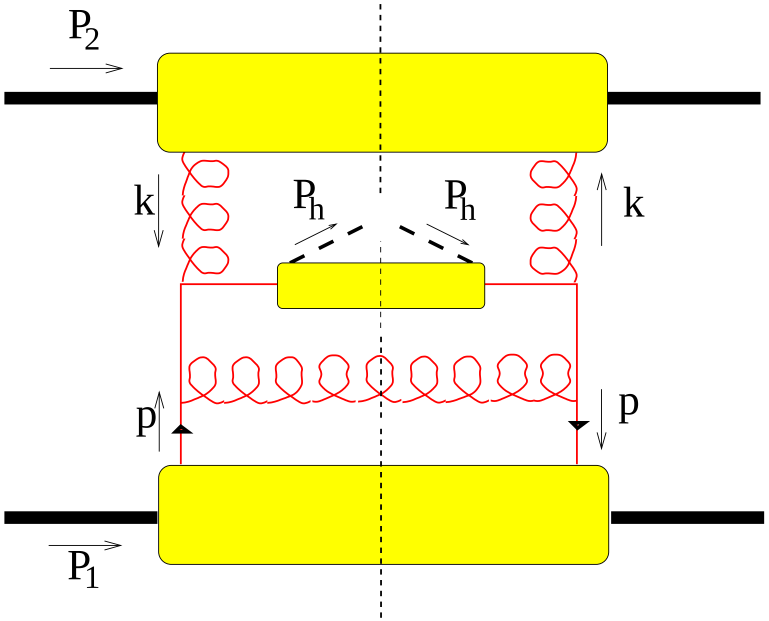
TMD factorization of $p^{\uparrow} + p \to \pi + X$ not established (power suppressed asymmetry), but it works phenomenologically

Anselmino *et al.*, since '95

Single spin asymmetries in $p^{\uparrow}p \to \pi X$

Large (left-right) SSA have been observed in $p^{\uparrow}\,p \to \pi\,X$ [E704, AGS, STAR, BRAHMS]





However, twist-3 factorization not proven and would only apply at high p_T of the pion

Collinear factorization, rather than TMD factorization

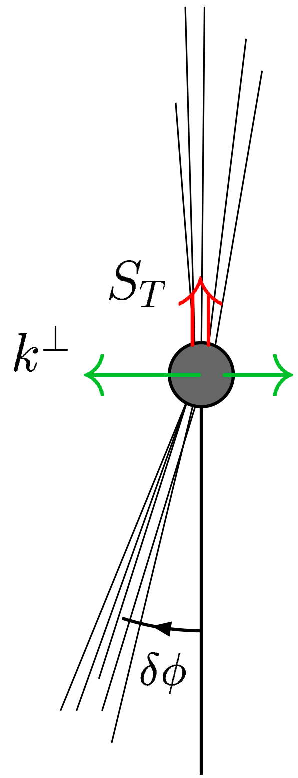
Expected to be a $\sin\phi_S$ asymmetry, but not demonstrated yet!

Sivers effect in dijet production

Asymmetric jet or hadron correlations in $p^{\uparrow} p \to h_1 \, h_2 \, X$

D.B. & Vogelsang '04

Bacchetta *et al.* '05



Sivers effect $\Rightarrow \sin \delta\phi$ asymmetry

$\delta\phi =$ dijet imbalance angle

RHIC data consistent with zero at the few percent level

STAR Collaboration, Abelev *et al.* '07

Theoretically this Sivers asymmetry is not as straightforward as in SIDIS or DY

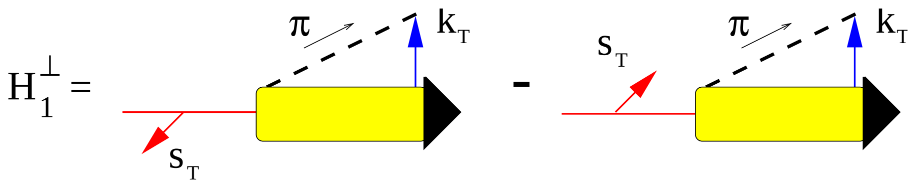
Problems with TMD factorization, except for the $P_{\perp} \sin \delta\phi$ weighted asymmetry

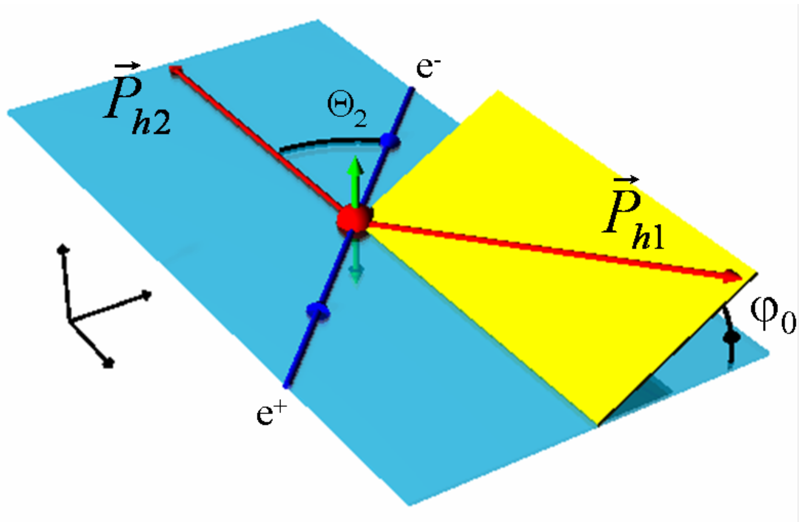
Collins & Qiu '07; Collins '07; Rogers & Mulders '10

Collins asymmetry in electron-positron annihilation

Collins function $H_1^\perp$ yields a $\cos 2\phi$ asymmetry in $e^+ \, e^- \to \pi^+ \, \pi^- \, X$

D.B., Jakob & Mulders, NPB '97





This measurement has been performed by the BELLE Collaboration

R. Seidl *et al.*, BELLE Collaboration, PRL '06; PRD '08

This allowed for the first extraction of transversity using SIDIS data

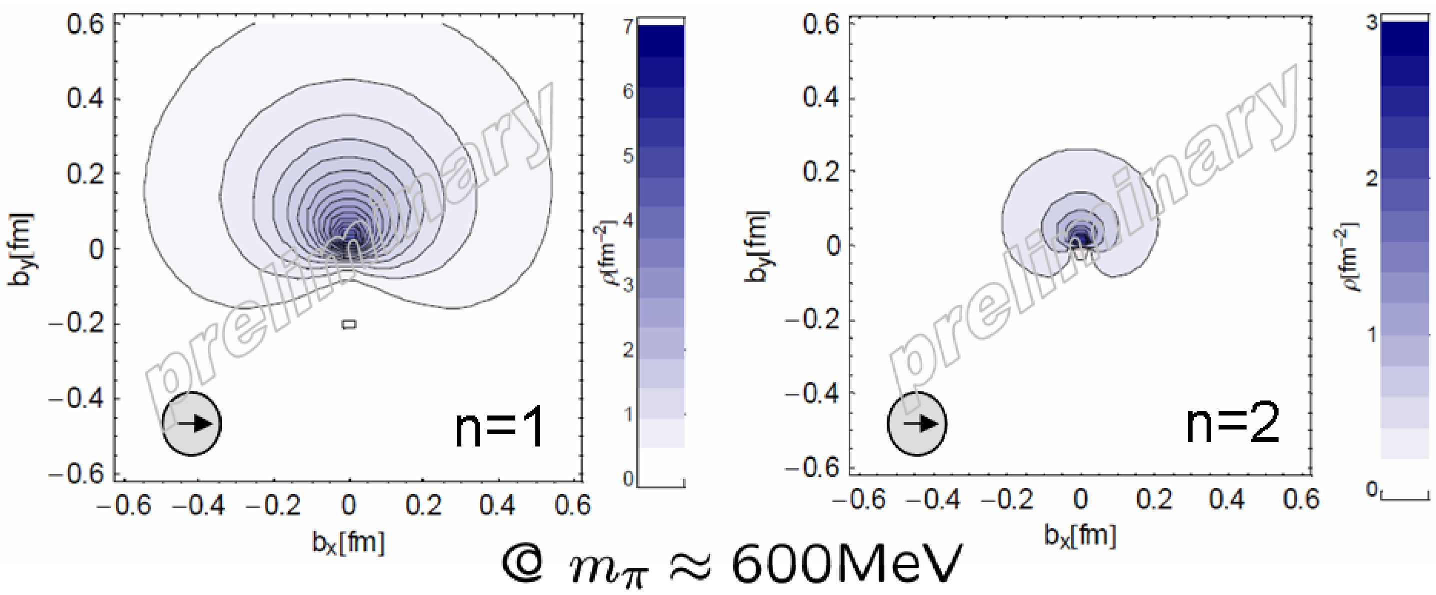
Anselmino *et al.*, PRD '07

Indications from lattice QCD calculations

$h_1^\perp$ may be related to an asymmetric spatial (impact parameter) distribution of transversely polarized quarks inside an unpolarized hadron

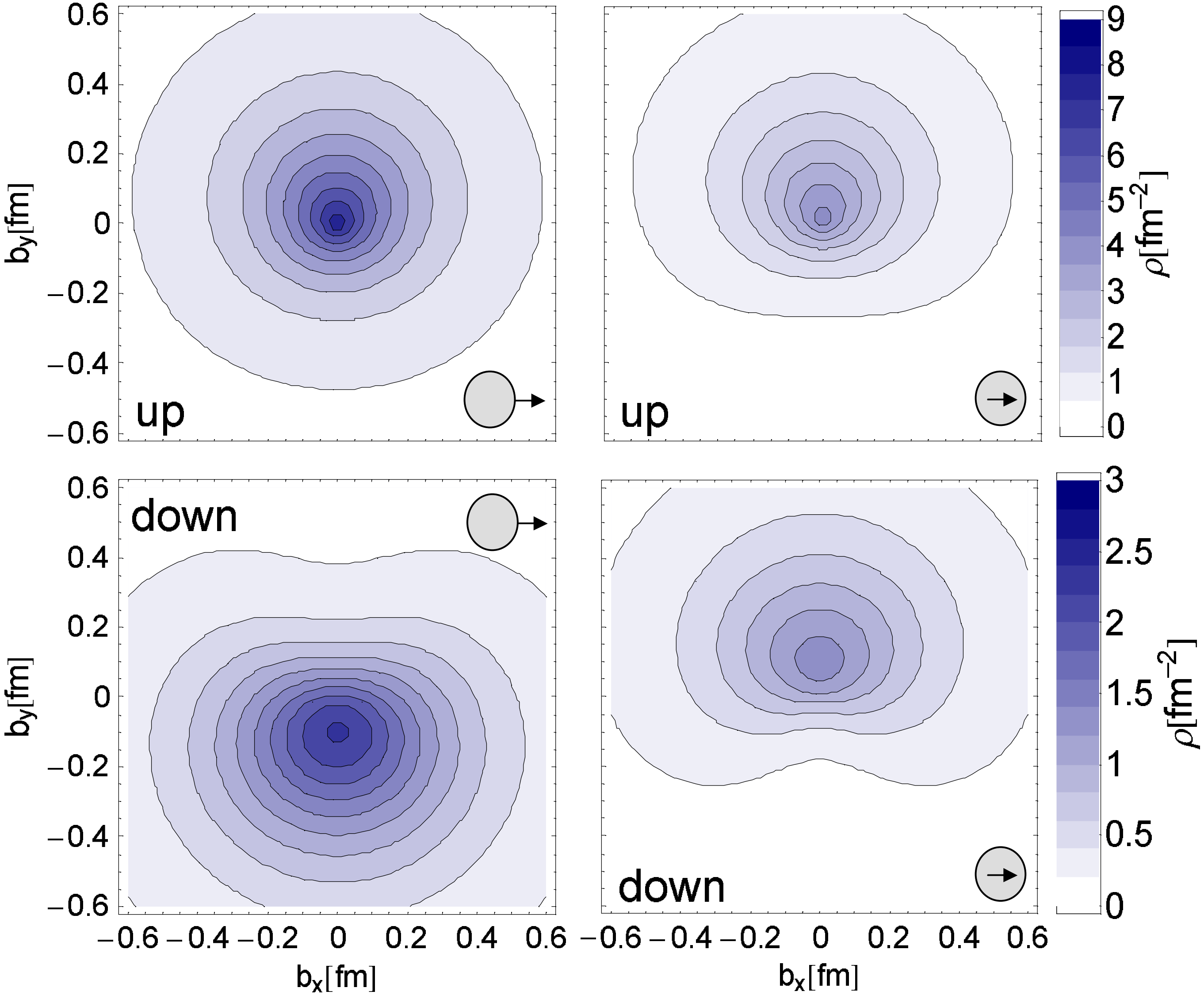
Burkardt, NPA 635 (2004) 185

Hägler recently showed at DIS 2007 the first lattice results relevant for this distribution inside the pion:



There is a clear $b_T \times s_T$ correlation inside a pion

Lattice QCD results



$h_1^{\perp\, u}$ same sign as $h_1^{\perp\, d}$

Burkardt & Hannafious '07

Also $S_T \times b_\perp$ correlations

QCDSF & UKQCD Collaboration, Göckeler *et al.* '07

Testing the TMD framework

In collinear factorization, pdf's and FF's are universal

In TMD factorization this no longer holds true, except for the TMD FFs

Metz '02; Collins & Metz '04; Yuan '08; Gamberg, Mukherjee & Mulders' 08; Meissner & Metz '09

TMDs can be *process dependent* or *nonuniversal*

When TMD factorization applies, this is a calculable dependence that can be tested

At present no single TMD has been reliably extracted from two different processes

Many asymmetries are to be measured still and also the *scale dependence* allows tests of the formalism within individual processes

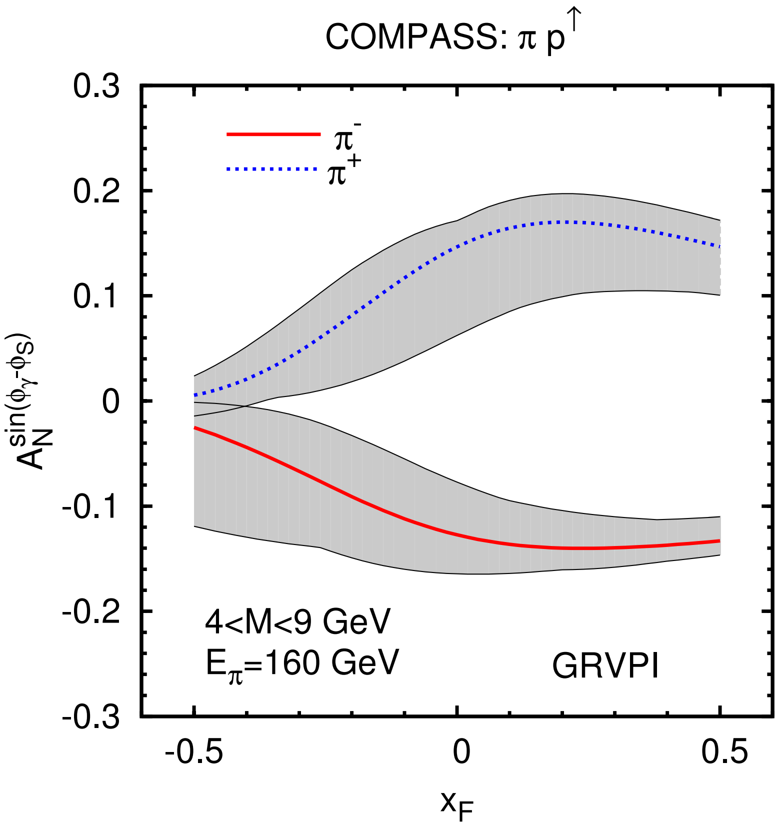
To be discussed:

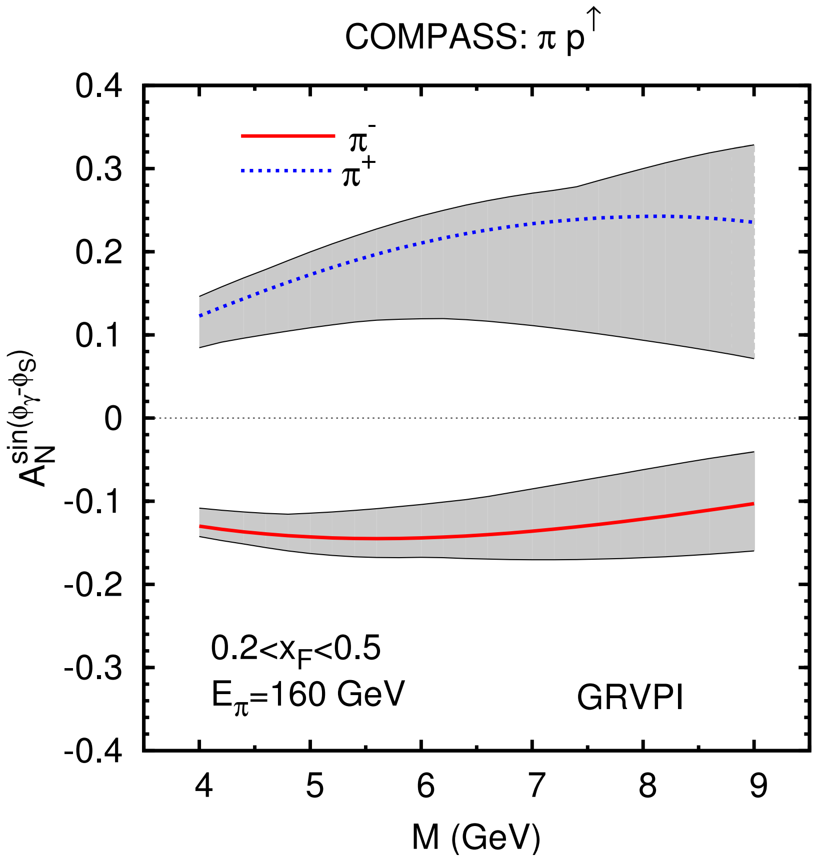
- Sivers function sign relation test
- $h_1^\perp$ in hadron collisions beyond DY

Sivers effect in Drell-Yan

Sivers effect also leads to a $\sin(\phi - \phi_S)$ asymmetry in Drell-Yan $\propto f_{1T}^{\perp} \bar{f}_1$

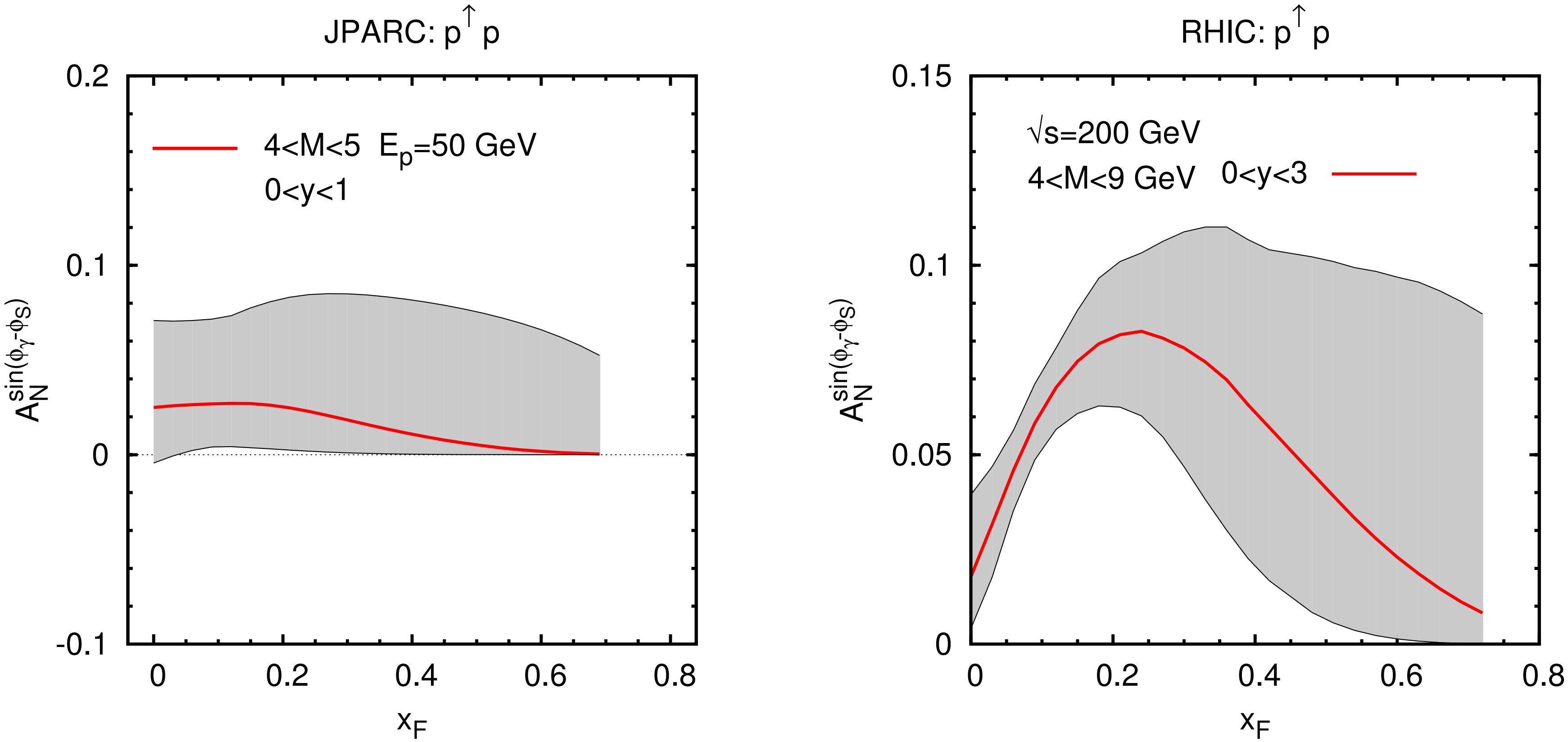
Some predictions based on fit to SIDIS data:





Anselmino *et al.* '09

Sivers effect in Drell-Yan



Anselmino *et al.* '09

$p^{\uparrow}p$ DY studies kinematically largely complementary to SIDIS data

These predictions take into account the *process dependence* of the Sivers function

Link structure of TMDs

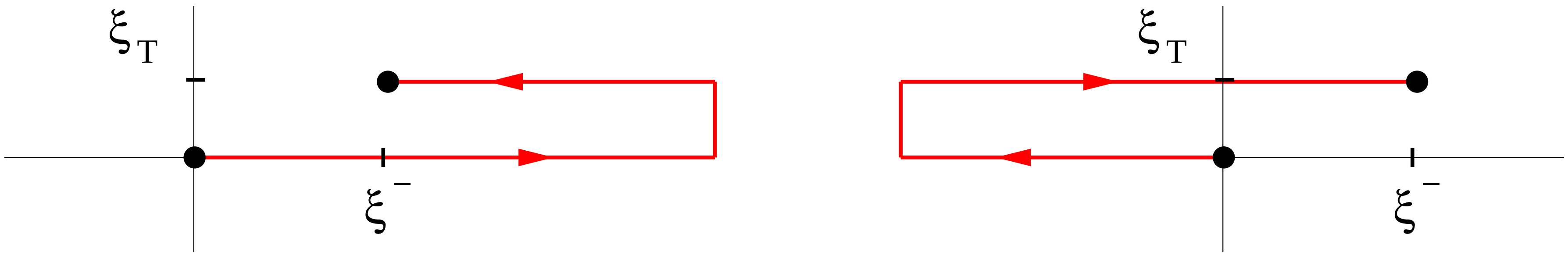
The gauge invariant definition of the Sivers function is not unique:

$$P\cdot(\boldsymbol{k}_T \times \boldsymbol{S}_T)\, f_{1T}^{\perp[\mathcal{C}]}(x,\boldsymbol{k}_T^2) \propto \text{F.T.}\, \langle P,S_T|\overline{\psi}(0)\, \mathcal{L}_{\mathcal{C}}[0,\xi]\, \gamma^+\, \psi(\xi)|P,S_T\rangle\big|_{\xi=(\xi^-,0^+,\boldsymbol{\xi}_T)}$$

Wilson line along contour $\mathcal{C}$:

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

Gauge invariant definition of TMDs in SIDIS contains a future pointing Wilson line (FSI), whereas in Drell-Yan (DY) it is past pointing (ISI)



Belitsky, Ji & Yuan '03

Process dependence of the Sivers function

Time reversal invariance relates the Sivers functions of SIDIS and Drell-Yan

This is a *calculable* process dependence, which yields the relation (Collins '02):

$$f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]} \qquad \text{to be tested}$$

Ignoring the link dependence yields $f_{1T}^{\perp} = 0$ because of time reversal invariance
Sivers function is naive T-odd (since odd under T when not exchanging ISI and FSI)

The more hadrons are observed, the more complicated the end result (ISI and FSI)

Bomhof, Mulders & Pijlman '04

This leads to trouble for processes like $p\,p \to$ jet jet X
TMD factorization fails for such processes, except for certain weighted asymmetries

Collins & Qiu '07; Collins '07; Rogers & Mulders '10

This does *not* cast doubt on the above sign relation

Sivers effect in dijet production

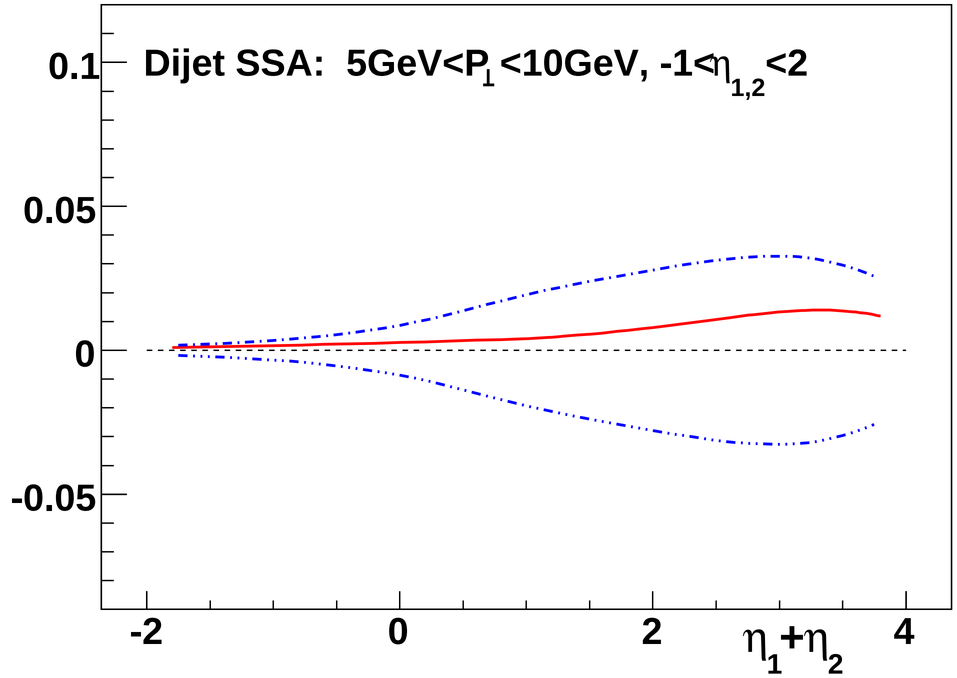
Asymmetric jet correlations:

$p^{\uparrow}\, p \to \text{jet jet}\, X \qquad \propto f_{1T}^{\perp}$

This Sivers function is not just related to SIDIS Sivers function by a sign

Bacchetta, Bomhof, Mulders & Pijlman '05

Bomhof, Mulders, Vogelsang & Yuan '07



Problems with TMD factorization, except for the $P_{\perp} \sin \delta\phi$ weighted asymmetry

Overall sign relation test

For the experimental test it is crucial the functions are probed at the same x and k_T^2

$$f_{1T}^{\perp[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp[\mathrm{DY}]}(x, k_T^2)$$

The function may have a Q^2 dependent node as a function of x and/or k_T

D.B. '11; Kang, Qiu, Vogelsang, Yuan '11

DY experiments should include data around and below the J/ψ and cover $x \leq 0.35$

The node can be at different places for different flavors, although one expects:

$$f_{1T}^{\perp u}(x, k_T^2) = -f_{1T}^{\perp d}(x, k_T^2) + \mathcal{O}(N_c^{-1})$$

Pobylitsa '03; Drago '05

Some model calculations show a node, but not for all flavors

d: Lu, Ma, NPA 741 (2004) 200; Courtoy, Fratini, Scopetta, Vento, PRD 78 (2008) 034002

u: Bacchetta, Conti, Radici, PRD 78 (2008) 074010

Sign change of $h_1^\perp$

How about measuring the overall sign change of $h_1^\perp$?

$$h_1^{\perp[\mathrm{SIDIS}]}(x,k_T^2) = -h_1^{\perp[\mathrm{DY}]}(x,k_T^2)$$

Chiral-odd functions always appear in pairs, hence not straightforward

If one restricts to valence quarks and assumes up-quark dominance, it is possible

One has to measure asymmetries in the following processes to fix the sign change

- $(e\,p^\uparrow \to e'\,h\,X)/(e\,p \to e'\,h\,X) \propto h_1^u/h_1^{\perp\,u\,[\mathrm{SIDIS}]}$
- $(\pi^-\,p^\uparrow \to \ell\bar{\ell}X)/(\pi^-\,p \to \ell\bar{\ell}X) \propto h_1^u/h_1^{\perp\,u\,[\mathrm{DY}]}$ (or $\bar{p}\,p^\uparrow \to \ell\bar{\ell}X \propto h_1^{\perp\,u\,[\mathrm{DY}]}h_1^u$)

h_1: distribution of transversely polarized quarks inside transversely polarized hadrons

Including d-quarks requires many more observables, using e^+e^- and $p\,p$ collisions and exploiting $\Lambda^\uparrow$ and dihadron (IFF) final states, forming in principle a closed system

Future experiments relevant for $h_1^\perp$ studies

Future DY data

E906/SeaQuest (Fermilab), RHIC, NICA (JINR, Dubna), JPARC: $p\,p$ collisions

COMPASS (CERN): $\pi\,p$ scattering

GSI: $\bar{p}\,p$ collisions

Future SIDIS data

COMPASS, JLab 12 GeV upgrade, EIC?

For $\cos(2\phi)$ in SIDIS, data at typical Q^2 of DY would be preferred

Other possible hadronic collision data

Tevatron, LHC

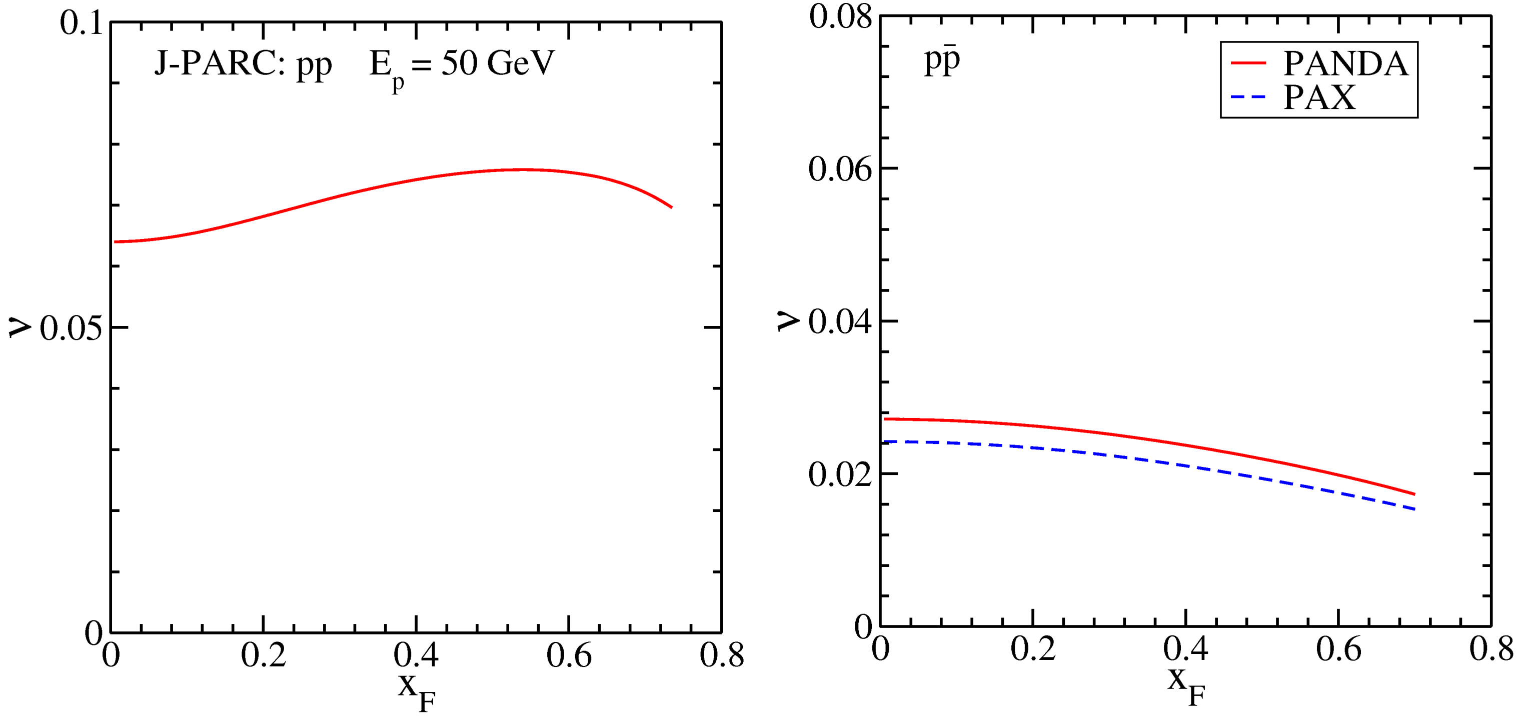
Future unpolarized DY data

$\cos(2\phi)$ asymmetry for $p\,p$ and $p\,d$ turns out to be small

FNAL-E866/NuSea Collaboration, L.Y. Zhu *et al.* '07 & '09

Asymmetry for $\bar{p}\,p$ expected to be very similar to $\pi\,p$ (both have valence antiquarks)

Although this depends on the kinematics too of course:



Lu & Schmidt '09

$h_1^\perp$ in $p\,p$ or $p\,\bar{p} \to \gamma$ jet X

$$\frac{d\sigma^{h_1\,h_2\to\gamma\text{ jet }X}}{d\eta_\gamma\,d\eta_j\,d^2\boldsymbol{K}_{\gamma\perp}\,d^2\boldsymbol{q}_\perp} \quad\propto\quad \left(1+\nu_{\mathrm{DY}}\,R\cos 2(\phi_\perp-\phi_\gamma)\right)$$

ν_{DY} probed at the scale $|\boldsymbol{K}_{\gamma\perp}|$

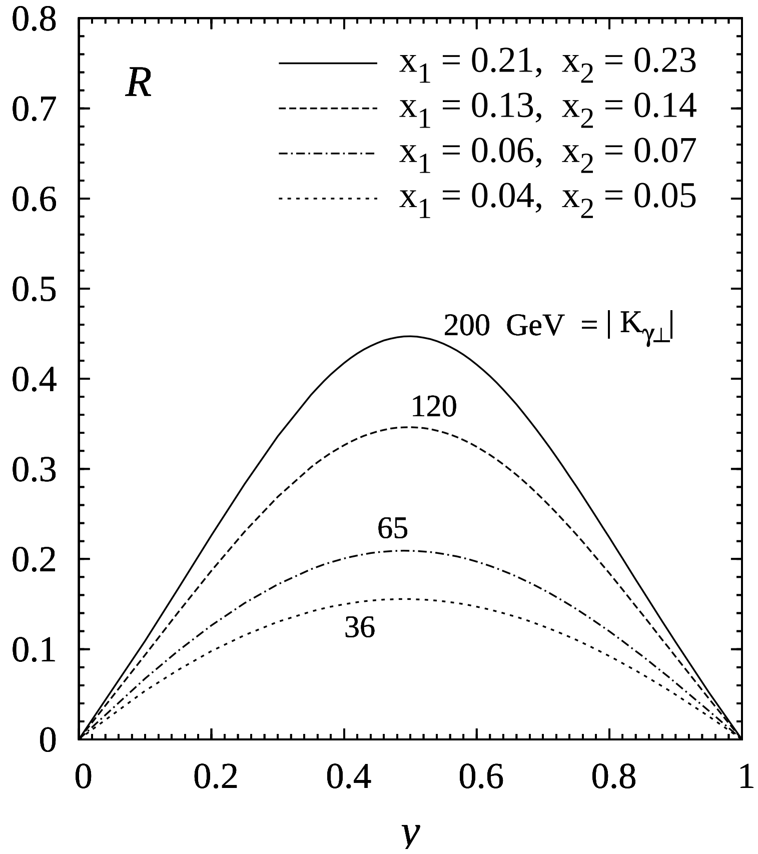
Proportionality factor R only function of f_1

$$y \equiv -\frac{\hat{t}}{\hat{s}} = \frac{1}{e^{\eta_\gamma-\eta_j}+1}$$

For typical Tevatron kinematics in the central region (DØ, arXiv:0804.1107)

$\nu_{\mathrm{DY}}\,R \sim 5-15\%$ expected

D.B., Mulders & Pisano '08



At RHIC this will be around or below 1%, based on $\nu_{\mathrm{DY}}(pp)$ at $\sqrt{s}=40$ GeV

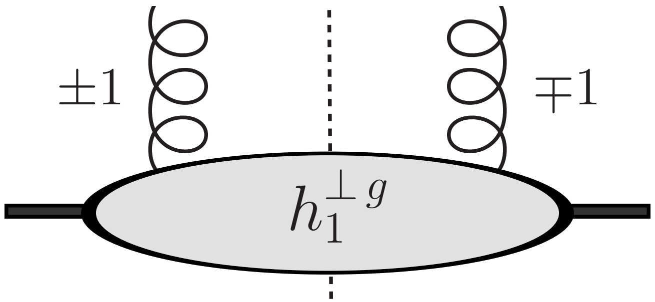
Linearly polarized gluons

Unpolarized protons can contain linearly polarized gluons

Mulders, Rodrigues '01

$h_1^{\perp\, g}$: linearly polarized gluons inside an unpolarized hadron (T-, chiral- & $\boldsymbol{k}_T$-even)

It is an interference between ± 1 helicity gluon states



T-even TMD, nevertheless sensitive to ISI/FSI, hence in principle process dependent

Linearly polarized gluons are also generated perturbatively

Catani, Grazzini '10

$h_1^\perp$ in dijet production

$h_1^\perp$ of quarks *and gluons* contribute to the dijet imbalance $\delta\phi$ distribution
Lu & Schmidt '08; D.B., Mulders & Pisano '09

$h_1^{\perp\, g}$: linearly polarized gluons inside an unpolarized hadron (T-, chiral- & $\boldsymbol{k}_T$-even)

In the plane transverse to the collision axis: $\delta\phi = \phi_{j_1} - \phi_{j_2} - \pi$

In unpolarized hadron scattering its distribution is often used to extract $\langle k_T^2 \rangle$ of partons

Sizeable $h_1^\perp$ contributions can modify the $\delta\phi$ distribution (especially in $p\,\bar{p} \to$ jet jet X)

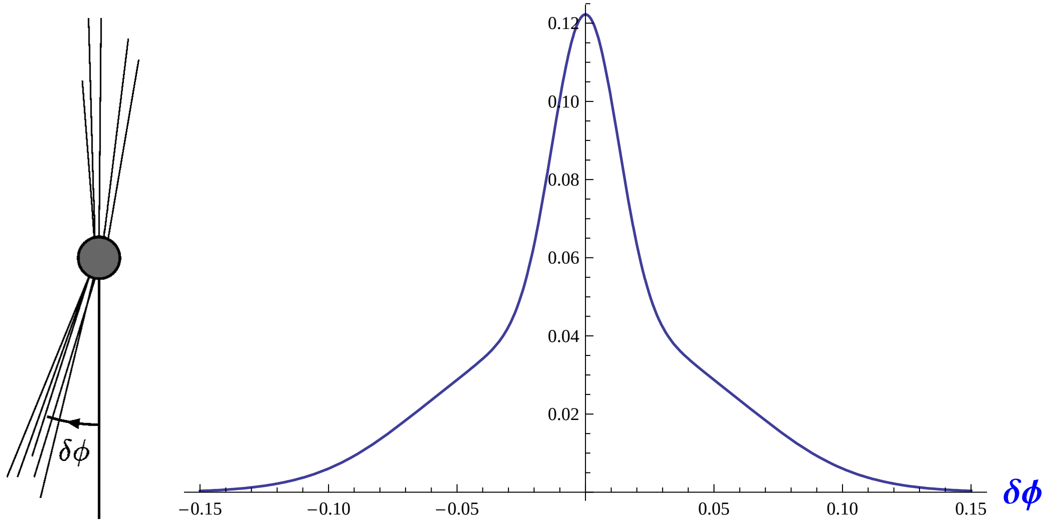


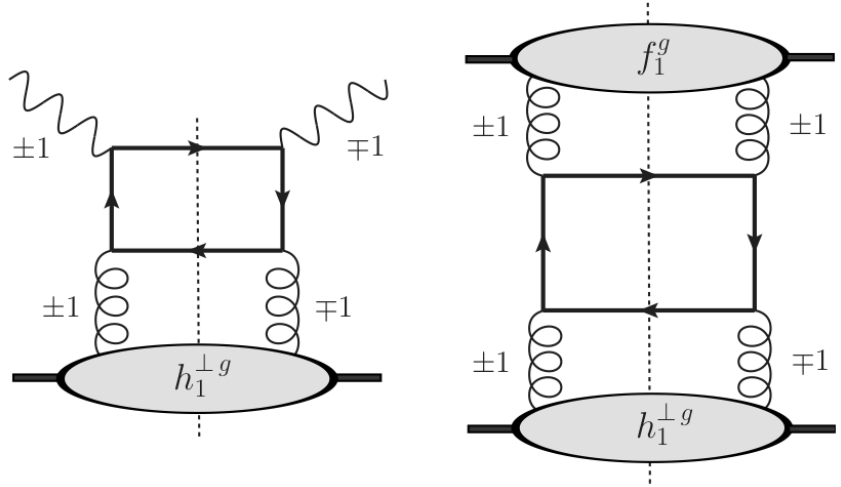
Illustration of a possible modification by $h_1^\perp$

$h_1^{\perp g}$ in heavy quark production

New ways to experimentally probe this distribution in heavy quark pair production

$\cos 2(\phi_T - \phi_\perp)$ asymmetries

$\phi_{T/\perp}$: angles of $K_\perp^Q \pm K_\perp^{\bar{Q}}$



D.B., Brodsky, Mulders, Pisano, PRL '11

Problems regarding factorization in $p\,p$ (RHIC, LHC) or $p\,\bar{p}$ (Tevatron) case

Rogers, Mulders '10

Therefore, best done at an EIC or LHeC

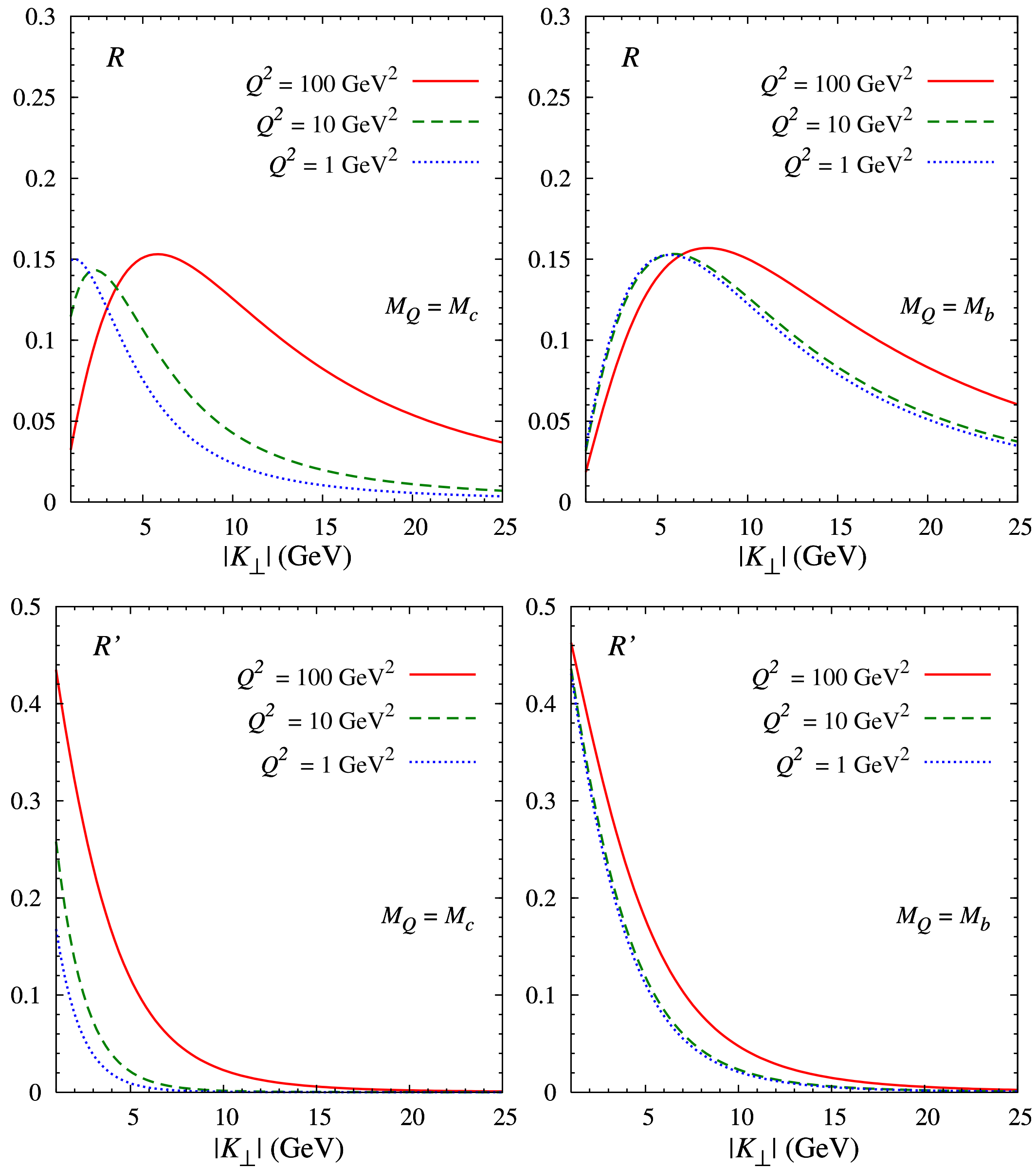
Or consider $h_1^\perp$ in $p\,p$ or $p\,\bar{p} \to \gamma\,\gamma\,X$ (theoretically safe, but percent level)

Qiu, Schlegel, Vogelsang '11

Maximum asymmetries in heavy quark production

R = bound on $|\langle\cos 2(\phi_T - \phi_\perp)\rangle|$

R' = bound on $|\langle\cos 2(\phi_\ell - \phi_T)\rangle|$



D.B., Brodsky, Mulders, Pisano
arXiv:1107.1400

$h_1^{\perp g}$ in $p\,p$ or $p\,\bar{p} \to H\ X$

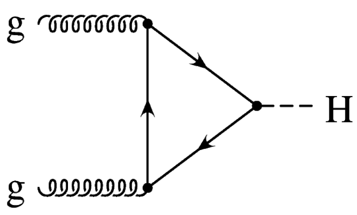
Linearly polarized gluons enter Higgs production ($\sigma(Q_T)$) at NNLO pQCD

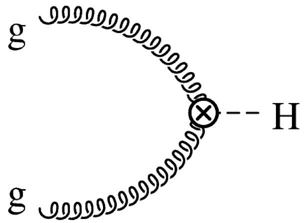
Catani, Grazzini '10

A nonperturbative distribution of linearly polarized gluons ($h_1^{\perp g}$) can be present too

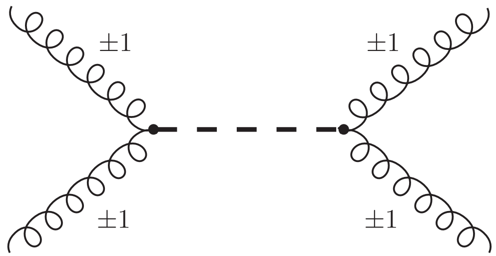
Can affect (pseudo-)scalar Higgs production at low transverse momentum

Higgs production happens mainly through $gg \to H$

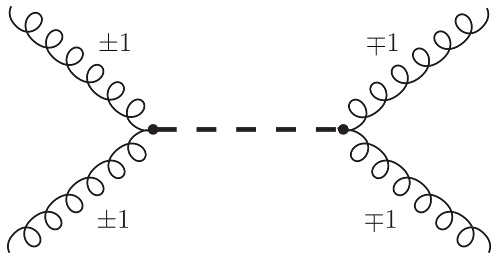




$\sigma \propto |\mathcal{M}|^2 \propto$

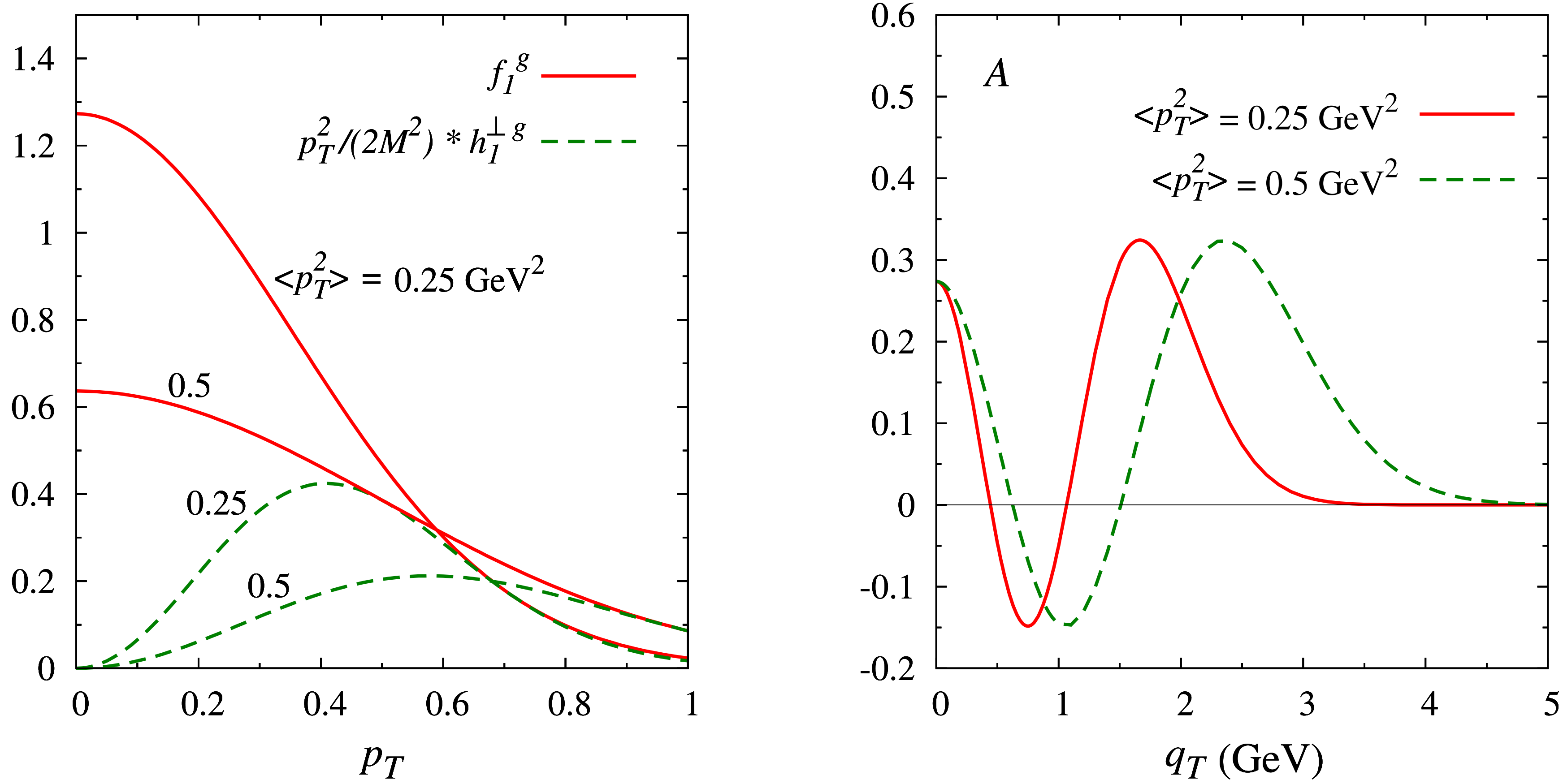


$+$



$h_1^{\perp g}$ in $p\,p$ or $p\,\bar{p} \to H\ X$

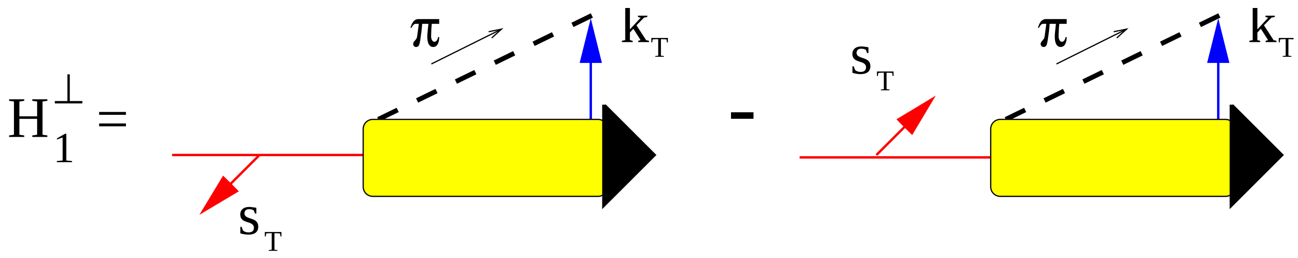
ϕ-independent cross section is of the form: $1 \pm \mathcal{A}[h_1^{\perp g} h_1^{\perp g}]$ (+ for H^0; − for A^0)



D.B., Pisano, Schlegel, Vogelsang, *in preparation* (including backgrounds γZ, WW, ZZ)

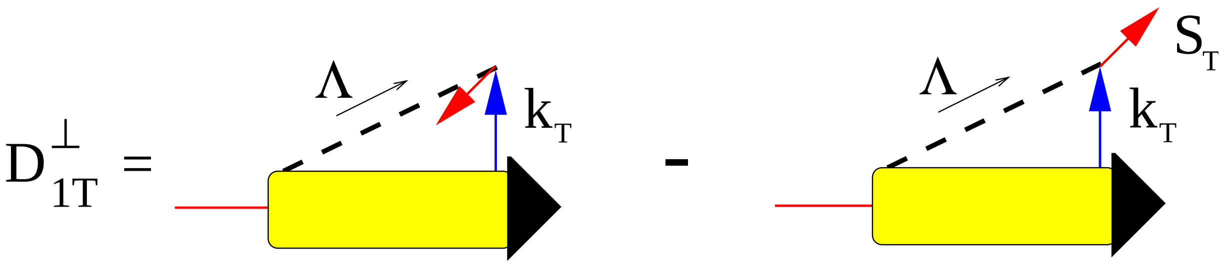
Spin effects in fragmentation

Collins effect



Collins '93

Polarizing fragmentation function

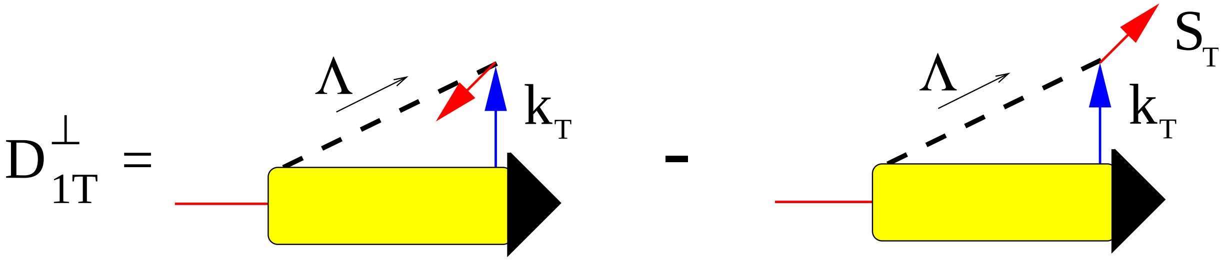


Mulders & Tangerman '96

Both these fragmentation functions *are* universal according to recent insights

Metz '02; Collins & Metz '04; Yuan '08; Gamberg, Mukherjee & Mulders' 08; Meissner, Metz '09

T-odd fragmentation functions



Λ polarization arises in the fragmentation of an *unpolarized* quark

Hence, the name "polarizing fragmentation function"

$D_{1T}^{\perp}$ has been extracted from fixed target $p + p(Be) \to \Lambda^{\uparrow}(\bar{\Lambda}^{\uparrow}) + X$ data

Anselmino, D.B., D'Alesio & Murgia, 2001

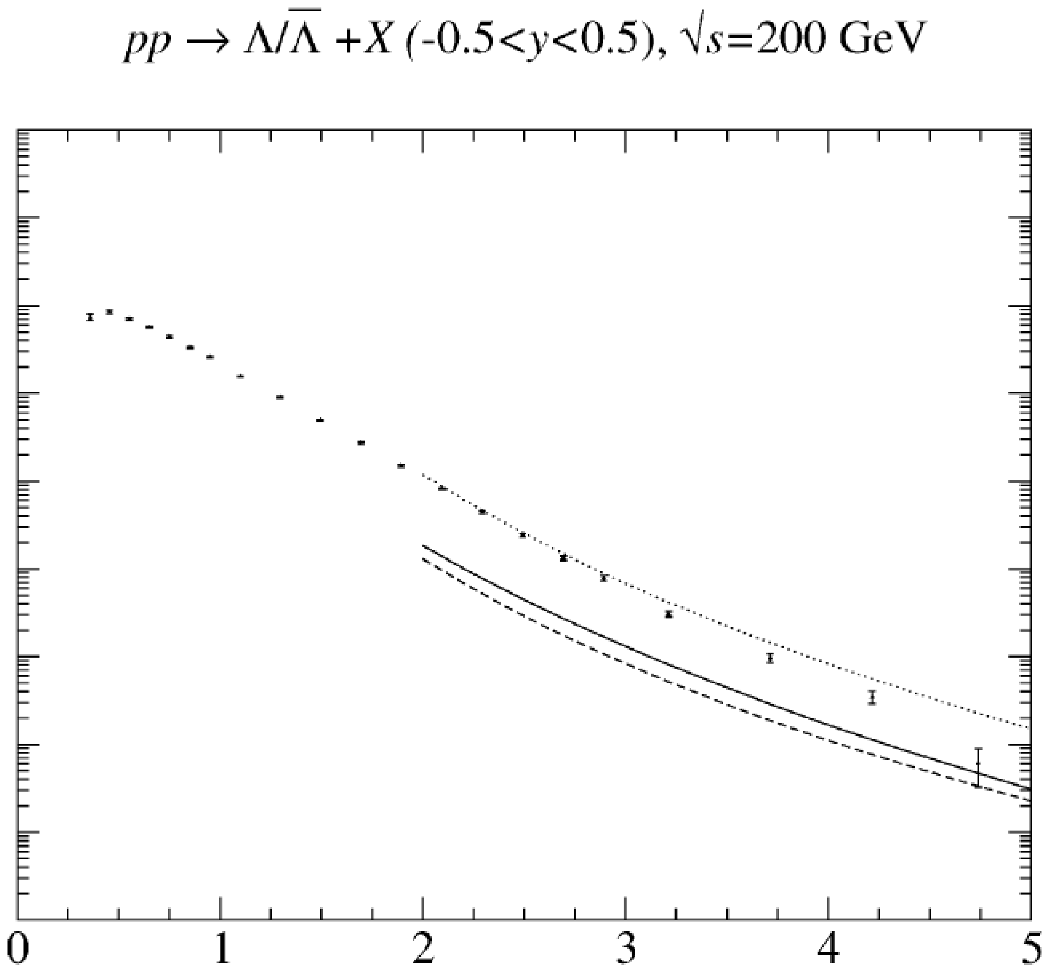
However, this process is not TMD factorizing (like $p^{\uparrow}p \to \pi X$)

Nevertheless, reasonable functions were obtained (u,d is opposite in sign to s)

Extraction used old Λ fragmentation functions

Newer functions (AKK) pose a problem

Λ fragmentation function problem



p_T distribution

solid: AKK08

dotted: AKK

dashed: DSV

data: STAR

"a possible inconsistency between the pp and e^+e^- reaction data for $\Lambda/\bar{\Lambda}$ production"

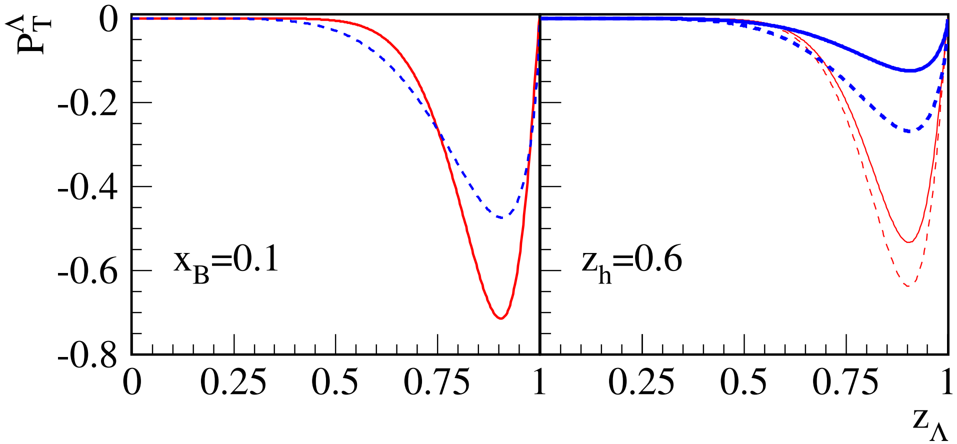
AKK, NPB 803 (2008) 42

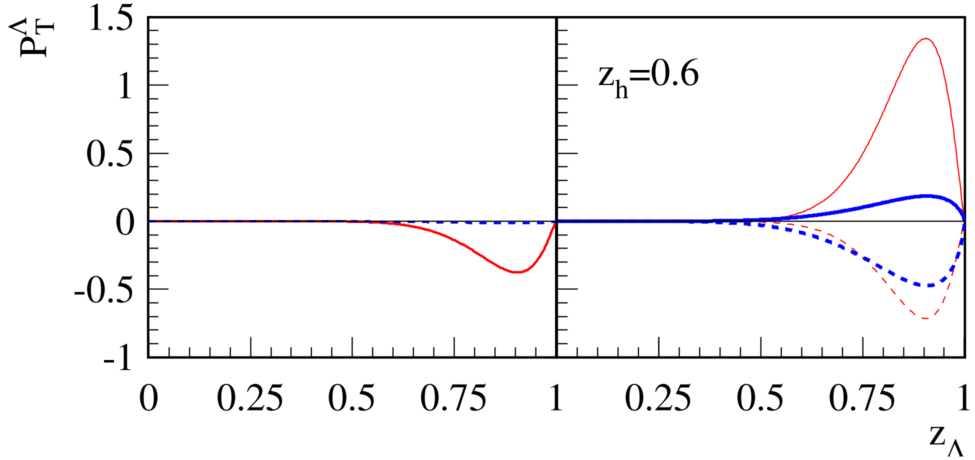
Test of universality of $D_{1T}^{\perp}$

Comparison of $ep \to e' \Lambda^{\uparrow} X$ and $e^+e^- \to \Lambda^{\uparrow} X$ can be used to test universality of $D_{1T}^{\perp}$

However, $e^+e^- \to \Lambda^{\uparrow} X$ is very sensitive to cancellations between u, d and s contributions

A better test is via $e^+e^- \to \Lambda^{\uparrow}\, \pi$ (or K) X to impose flavor selection





D.B., Kang, Vogelsang,Yuan, PRL 2010

Fig 1: SIDIS, SU(3)-symmetric (solid) and broken (dashed) spin-averaged FFs

Fig 2: $e^+e^- \to \pi^{\pm} + \Lambda^{\uparrow} + X$, SU(3)-symmetric (thin) and broken (thick), solid/dashed is $\pi^{\pm}$

Fig 3: $e^+e^- \to$ jet $+ \Lambda^{\uparrow} + X$, SU(3)-symmetric (solid) and broken (dashed) spin-averaged FFs

Fig 4: $e^+e^- \to K^{\pm} + \Lambda^{\uparrow} + X$, SU(3)-symmetric (thin) and broken (thick), solid/dashed is $K^{\pm}$

Universality of $D_{1T}^{\perp}$

What about using $p\,p$ collisions at RHIC or LHC?

Capabilities to measure Λ polarization via $\Lambda \to p\,\pi^-$ are usually restricted to the midrapidity region, where the degree of transverse polarization is very small

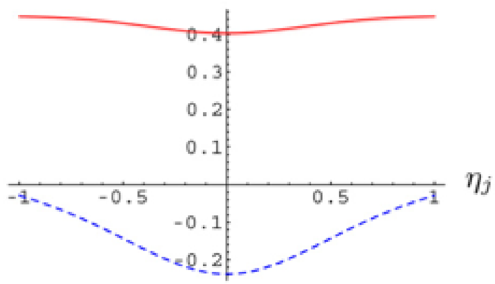
$P_\Lambda = 0$ at $\eta = 0$ in $p\,p \to \Lambda^\uparrow X$ collisions in cms

Alternative: consider jet+Λ production: $p\,p \to \left(\Lambda^\uparrow \text{jet}\right)\ \text{jet}\, X$

Such an asymmetry does not need to vanish at $\eta = 0$

D.B., Bomhof, Hwang & Mulders, PLB 659 (2008) 127; D.B., 0907.1610

Allows a test of universality at RHIC and LHC through $\hat{t}/\hat{s}$ dependence, but not too distinctive



Universal $D_{1T}^{\perp}$ in $p\,p \to (\Lambda^{\uparrow}\text{jet})\ X$

Assuming universality of $D_{1T}^{\perp}$ one can consider $p\,p \to (\Lambda^{\uparrow}\text{jet})\ X$ if $E_{\text{jet}}/\sqrt{s} \ll 1$

D.B., 1007.3145

Assuming universality, the away-side jet is not needed, allowing to study a rather simple asymmetry *in the lab frame* $\propto \mathbf{K}_j\cdot(\mathbf{K}_\Lambda\times\mathbf{S}_\Lambda)$ with analyzing power:

$$\frac{\int \frac{dy}{y} \sum_q \left(f^{qg} + f^{gq}\right) d\hat{\sigma}_{qg} D_{1T}^{\perp\, q}}{\int \frac{dy}{y} \left[\sum_q \left(f^{qg} + f^{gq}\right) d\hat{\sigma}_{qg} \left(D_1^q + D_1^g\right) + f^{gg} d\hat{\sigma}_{gg} D_1^g\right]}$$

where $f^{ab} \equiv x_1 f_1^a(x_1) x_2 f_1^b(x_2)$, $d\hat{\sigma}_{ab} \equiv d\hat{\sigma}_{ab\to ab}(y) + d\hat{\sigma}_{ab\to ab}(1-y)$ with $y = -\hat{t}/\hat{s}$

For further details see: D.B., 1007.3145

Needs small ratio $E_{\text{jet}}/\sqrt{s}$ though

Test of TMD factorization through scale dependence

TMD factorization dictates the Q^2 dependence of azimuthal asymmetries

TMD or Collins-Soper factorization of the hadron tensor of DY:

$$W_{\mathrm{DY}}^{\mu\nu} \propto \left|H\big(x_1, x_2, Q^2\big)\right|^2 \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{l}_T - \boldsymbol{q}_T)$$

$$\times \operatorname{Tr}\left\{\Phi^a(x_1, \boldsymbol{p}_T)\gamma^\mu \bar{\Phi}^a(x_2, \boldsymbol{k}_T)\gamma^\nu\right\} U(l_T^2) \;+\; \mathcal{O}(Q_T^2/Q^2)$$

Collins & Soper '81; Ji, Ma & Yuan '04 & '05

In the perturbative region all ingredients are known to one-loop order

For the nonperturbative region parameters have been fitted to data

Allows prediction of the Q^2 dependence of azimuthal asymmetries

D.B. '01 & '09

Test of TMD factorization through scale dependence

Initial studies of the Q^2 dependence of azimuthal asymmetries indicate that:

	low Q_T	high Q_T
$\cos 2\phi$	$Q^{-0.9} - Q^{-1.0}$	Q^{-2}
Sivers	$Q^{-0.5} - Q^{-0.6}$	$f(1/\log Q^2)$

Collins, PRL 42 (1979) 291; D.B., NPB 603 (2001) 195 & NPB 806 (2009) 23

Kang, Qiu, PRD 79 (2009) 016003; Zhou, Yuan, Liang, PRD 79 (2009) 114022

Vogelsang, Yuan, PRD 79 (2009) 094010; Braun, Manashov, Pirnay, PRD 80 (2009) 114002

Tests of these dependences need large Q^2 range

For Sivers asymmetry best done at a future EIC

Other relevant theoretical studies:

Mert Aybat & Rogers, PRD 83 (2011) 114042; Kang, Xiao, Yuan, arXiv:1106.0266

Conclusions

- Sivers, BM & Collins effect asymmetries visible in the data
- No single TMD has been reliably extracted from two different processes yet
- Calculable process dependence, nonuniversality to be tested
- Opportunities for hadron colliders:
  - Check of overall sign relation of Sivers effect in SIDIS and DY
  - Uncovering possible nodes
  - $h_1^{\perp q}$ and $h_1^{\perp g}$ studies in $\gamma\gamma$, γj, jj, $Q\bar{Q}$, H^0, A^0, γZ, WW, ZZ production
  - Λ polarization ($D_{1T}^{\perp}$) studies (first measurement, universality)
  - Scale dependence studies before EIC
- All in all, a lot of theoretical & experimental progress in recent years, but the TMD picture is far from fully explored